Mathematical Constants

1. $\pi = 3.14159$
2. $\pi^2 = 9.8696$
3. $\sqrt{\pi} = 1.7724$
4. $\frac{1}{\pi} = 0.3183$
5. $e = 2.7182$
6. $\log_{10} \pi = 0.4971$
7. $\log_{10} e = 0.4343$
8. $\log_e 10 = 2.3026$
9. Plank's Constants = $6.624 \times 10^{-27}\, erg.Sec$
10. Avogadro's number = $6.023 \times 10^{23}\, mole$
11. Constant of gravitation = $6.670 \times 10^{-8}\, dyne.cm^2 / gram^2$
12. $g = 32.16\, ft/\sec^2 = 980\, cm/\sec^2$

Basic Derivative Results

If $y = f(x)$, then

1) $\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

2) $f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

3) $L.H.D = f'(a-0) = \lim_{h \to 0-0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$

4) $R.H.D = f'(a+0) = \lim_{h \to 0+0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

General Derivative Formulas:

1) $\frac{d}{dx}(c) = 0$ Where c is any constant.

2) $\frac{d}{dx} x^n = nx^{n-1}$ It is called Power Rule of Derivative.

3) $\frac{d}{dx}x = 1$

4) $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}\frac{d}{dx}f(x)$ Power Rule for Function.

5) $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

6) $\frac{d}{dx}\sqrt{f(x)} = \frac{1}{2\sqrt{f(x)}}\frac{d}{dx}f(x) = \frac{1}{2\sqrt{f(x)}}f'(x)$

7) $\frac{d}{dx}c \cdot f(x) = c\frac{d}{dx}f(x) = c \cdot f'(x)$

8) $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x) = f'(x) \pm g'(x)$

9) $\frac{d}{dx}[f(x) \cdot g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$ It is called Product Rule.

10) $\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$ It is called Quotient Rule.

Derivative of Logarithm Functions:

11) $\frac{d}{dx}\ln x = \frac{1}{x}$

12) $\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$

13) $\frac{d}{dx}\ln f(x) = \frac{1}{f(x)}\frac{d}{dx}f(x)$

14) $\frac{d}{dx}\log_a f(x) = \frac{1}{f(x)\ln a}\frac{d}{dx}f(x)$

Derivative of Exponential Functions:

15) $\frac{d}{dx}e^x = e^x$

16) $\frac{d}{dx}e^{f(x)} = e^{f(x)}\frac{d}{dx}f(x)$

17) $\frac{d}{dx}a^x = a^x \ln a$

18) $\frac{d}{dx}a^{f(x)} = a^{f(x)} \ln a \frac{d}{dx} f(x)$

19) $\frac{d}{dx}x^x = x^x(1 + \ln x)$

Derivative of Trigonometric Functions:

20) $\frac{d}{dx}Sinx = Cosx$

21) $\frac{d}{dx}Cosx = Sinx$

22) $\frac{d}{dx}Tanx = Sec^2x$

23) $\frac{d}{dx}Cotx = -Co\sec^2 x$

24) $\frac{d}{dx}Secx = Secx \cdot Tanx$

25) $\frac{d}{dx}Co\sec x = -Co\sec x \cdot Cotx$

Derivative of Hyperbolic Functions:

26) $\frac{d}{dx}Sinhx = Coshx$

27) $\frac{d}{dx}Coshx = Sinhx$

28) $\frac{d}{dx}Tanhx = Sech^2x$

29) $\frac{d}{dx}Cothx = -Co\sec h^2x$

30) $\frac{d}{dx} Sechx = -Sechx \cdot Tanhx$

31) $\frac{d}{dx} Ce\,\mathbf{sec}\,hx = -Co\,\mathbf{sec}\,hx \cdot Cothx$

Derivative of Inverse Trigonometric Functions:

32) $\frac{d}{dx} Sin^{-1}x = \frac{1}{\sqrt{1-x^2}}, \quad -1 < x < 1$

33) $\frac{d}{dx} Cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}, \quad -1 < x < 1$

34) $\frac{d}{dx} Tan^{-1}x = \frac{1}{1+x^2}$

35) $\frac{d}{dx} Cot^{-1}x = \frac{-1}{1+x^2}$

36) $\frac{d}{dx} Sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}, \quad |x| > 1$

37) $\frac{d}{dx} Co\,\mathbf{sec}^{-1}\,x = \frac{-1}{x\sqrt{x^2-1}}, \quad |x| > 1$

Derivative of Inverse Hyperbolic Functions:

38) $\frac{d}{dx} Sinh^{-1}x = \frac{1}{\sqrt{1+x^2}}$

39) $\frac{d}{dx} Cosh^{-1}x = \frac{1}{\sqrt{x^2-1}}$

40) $\frac{d}{dx} Tanh^{-1}x = \frac{1}{1-x^2}, \quad |x| < 1$

41) $\frac{d}{dx} Coth^{-1}x = \frac{1}{x^2-1}, \quad |x| > 1$

42) $\frac{d}{dx} Sech^{-1}x = \frac{-1}{x\sqrt{1-x^2}}, \quad 0 < x < 1$

43) $\frac{d}{dx} Co\,sech^{-1}x = \frac{-1}{x\sqrt{1+x^2}}, \quad x > 0$

Results of Higher Derivatives

1) $y_n = \frac{d^n}{dx^n}(ax+b)^m = \frac{m!}{(m-n)!} a^n (ax+b)^{m-n}$

2) $y_n = \frac{d^n}{dx^n} \frac{1}{(ax+b)} = \frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$

3) $y_n = \frac{d^n}{dx^n} \ln(ax+b) = \frac{(-1)^n (n-1)! a^n}{(ax+b)^n}$

4) $y_n = \frac{d^n}{dx^n} Sin(ax+b) = a^n Sin\left(ax+b+n\frac{\pi}{2}\right)$

5) $y_n = \frac{d^n}{dx^n} Cos(ax+b) = a^n Cos\left(ax+b+n\frac{\pi}{2}\right)$

6) $y_n = \frac{d^n}{dx^n} e^{ax} = a^n e^{ax}$

7) $y_n = \frac{d^n}{dx^n} e^{ax} Sin(ax+c) = \left(a^2+b^2\right)^{\frac{n}{2}} e^{ax} Sin\left(bx+c+nTan^{-1}\frac{a}{b}\right)$

8) $y_n = \frac{d^n}{dx^n} e^{ax} Cos(ax+c) = \left(a^2+b^2\right)^{\frac{n}{2}} e^{ax} Cos\left(bx+c+nTan^{-1}\frac{a}{b}\right)$

9) If $y = (ax+b)^n$, then $y_{n+r} = 0$ for $r > 0$

Leibniz's Theorem

10) $(f \cdot g)_n = f_n \cdot g + nf_{n-1} \cdot g_1 + \frac{n(n-1)}{2!} f_{n-2} \cdot g_2 + \cdots + f \cdot g_n$

Taylor's Theorem

11) $f(x+h) = f(x) + hf_1(x) + \frac{h^2}{2!} f_2(x) + \frac{h^3}{3!} f_3(x) + \cdots + \frac{h^n}{n!} f_n(x) + \cdots$

Meclaurin's Series

12) $f(x) = f(0) + xf_1(0) + \frac{x^2}{2!} f_2(0) + \frac{x^3}{3!} f_3(0) + \cdots + \frac{x^n}{n!} f_n(0) + \cdots$

Formulas of Integration

1) $\int 1 dx = x + c$

2) $\int a dx = ax + c$ Where a is any constant.

3) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

4) $\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c$

5) $\int \frac{1}{x} dx = \ln x + c$

6) $\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$

7) $\int a^x dx = \frac{a^x}{\ln x} + c$

8) $\int a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$

9) $\int e^x dx = e^x + c$

10) $\int e^{f(x)} dx = e^{f(x)} + c$

11) $\int af(x) dx = a \int f(x)$

12) $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$

13) $\int f(x) \cdot g(x) dx = f(x)\left(\int g(x) dx\right) - \left[f'(x)\left(\int g(x) dx\right)\right] dx$

14) $\int \ln x dx = x(\ln x - 1) + c$

15) $\int \sin x dx = -\cos x + c$

16) $\int \cos x dx = \sin x + c$

17) $\int \tan x dx = \ln \sec x + c$ **or** $-\ln \cos x + c$

18) $\int \cot x dx = \ln \sin x + c$

19) $\int \sec x dx = \ln(\sec x + \tan x) + c$ **or** $\ln \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) + c$

20) $\int \csc x dx = \ln(\csc x - \cot x) + c$ **or** $\ln \tan \frac{x}{2} + c$

21) $\int \sec^2 x dx = \tan x + c$

22) $\int \csc^2 x dx = -\cot x + c$

23) $\int \sec x \tan x dx = \sec x + c$

24) $\int \csc x \cot x dx = -\csc x + c$

25) $\int \sinh x dx = \cosh x + c$

26) $\int \cosh x dx = \sinh x + c$

27) $\int \tanh x dx = \ln \cosh x + c$

28) $\int \coth x dx = \ln \sinh x + c$

29) $\int \mathrm{sech} x dx = \tan^{-1}(\sinh x) + c$

30) $\int \mathrm{csch} x dx = -\coth^{-1}(\cosh x)$

31) $\int \mathrm{sech}^2 x dx = \tanh x + c$

32) $\int \mathrm{csch}^2 x dx = -\coth x + c$

33) $\int \mathrm{sech} x \tanh x dx = -\mathrm{sech} x + c$

34) $\int \mathrm{csch} x \coth x dx = -\mathrm{csch} x + c$

35) $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c$ **or** $\cos^{-1} \frac{x}{a} + c$

36) $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1} \frac{x}{a} + c$ **or** $\ln(x + \sqrt{x^2 - a^2}) + c$

37) $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1} \frac{x}{a} + c$ **or** $\ln(x + \sqrt{x^2 + a^2}) + c$

38) $\int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \tanh^{-1} \frac{x}{a} + c$ **or** $\frac{1}{2a} \ln\left(\frac{a + x}{a - x}\right) + c$

39) $\int \frac{1}{x^2-a^2}dx = -\frac{1}{a}\coth^{-1}\frac{x}{a}+c$ or $\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)+c$

40) $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

41) $\int \frac{1}{x\sqrt{a^2-x^2}}dx = -\frac{1}{a}\operatorname{sech}^{-1}\frac{x}{a}+c$ or $-\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)+c$

42) $\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\sec^{-1}\frac{x}{a}+c$

43) $\int \frac{1}{x\sqrt{x^2+a^2}}dx = -\frac{1}{a}\operatorname{csch}^{-1}\frac{x}{a}+c$ or $-\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)+c$

44) $\int \sqrt{a^2-x^2}dx = \frac{1}{2}x\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$

45) $\int \sqrt{x^2-a^2}dx = \frac{1}{2}x\sqrt{x^2-a^2}-\frac{a^2}{2}\cosh^{-1}\frac{x}{a}+c$ or $\frac{1}{2}x\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left(x+\sqrt{x^2-a^2}\right)+c$

46) $\int \sqrt{x^2+a^2}dx = \frac{1}{2}x\sqrt{x^2+a^2}-\frac{a^2}{2}\sinh^{-1}\frac{x}{a}+c$ or $\frac{1}{2}x\sqrt{x^2+a^2}+\frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)+c$

47) $\int e^{ax}\sin(bx+c)dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+c)+b\cos(bx+c)\right]$

48) $\int e^{ax}\cos(bx+c)dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos(bx+c)+b\sin(bx+c)\right]$

49) $\int \sin mx\cos nx dx = -\frac{\cos(m+n)x}{2(m+n)}-\frac{\cos(m-n)x}{2(m-n)}+c$

50) $\int \sin mx\sin nx dx = -\frac{\sin(m+n)x}{2(m+n)}+\frac{\sin(m-n)x}{2(m-n)}+c$

51) $\int \cos mx\cos nx dx = \frac{\sin(m+n)x}{2(m+n)}+\frac{\sin(m-n)x}{2(m-n)}+c$

52) $\int \sin^{-1}x dx = x\sin^{-1}x+\sqrt{1-x^2}+c$

53) $\int \cos^{-1}x dx = x\cos^{-1}x-\sqrt{1-x^2}+c$

54) $\int \tan^{-1} x\,dx = x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + c$

55) $\int \cot^{-1} x\,dx = x\cot^{-1} x + \frac{1}{2}\ln(1+x^2) + c$

56) $\int \sec^{-1} x\,dx = x\sec^{-1} x - \ln\left(x + \sqrt{x^2-1}\right) + c$

57) $\int \csc^{-1} x\,dx = x\csc^{-1} x + \ln\left(x + \sqrt{x^2-1}\right) + c$

58) $\int \frac{1}{a + b\sin x}dx = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\frac{a\tan^{-1}\frac{x}{2} + b}{\sqrt{a^2-b^2}}\right) + c$ if $a^2 > b^2$

59) $\int \frac{1}{a + b\sin x}dx = \frac{1}{\sqrt{a^2-b^2}}\ln\left(\frac{a\tan\frac{x}{a} + b - \sqrt{b^2-a^2}}{a\tan\frac{x}{a} + b + \sqrt{b^2-a^2}}\right) + c$ if $a^2 < b^2$

60) $\int \frac{1}{a + b\cos x}dx = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) + c$ if $a^2 > b^2$

61) $\int \frac{1}{a + b\cos x}dx = \frac{1}{\sqrt{a^2-b^2}}\ln\left(\frac{\sqrt{b+a} + \tan\frac{x}{2}\sqrt{b-a}}{\sqrt{b+a} - \tan\frac{x}{2}\sqrt{b-a}}\right) + c$ if $a^2 < b^2$

62) $\int \frac{1}{a + b\sinh x}dx = \frac{1}{\sqrt{a^2+b^2}}\ln\left(\frac{\sqrt{a^2+b^2} + a\tanh\frac{x}{2} - b}{\sqrt{a^2+b^2} - a\tanh\frac{x}{2} + b}\right) + c$

63) $\int \frac{1}{a + b\cosh x}dx = \frac{\sqrt{a+b} + \sqrt{a-b}\tanh\frac{x}{2}}{\sqrt{a+b} - \sqrt{a-b}\tanh\frac{x}{2}} + c$ if $a > b$

64) $\int \frac{1}{a + b\cosh x}dx = \frac{2}{\sqrt{b^2-a^2}}\tan^{-1}\sqrt{\frac{b-a}{b+a}}\tanh^{-1}\frac{x}{2} + c$ if $a < b$

Reduction Formulas of Integration

1) $\int \sin^n x dx = -\frac{\cos x \sin^{n-1} x}{n} + \frac{n-1}{n}\int \sin^{n-2} x dx$

2) $\int \cos^n x dx = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n}\int \cos^{n-2} x dx$

3) $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$

4) $\int \cot^n x dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx$

5) $\int \sec^n x dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x dx$

6) $\int \csc^n x dx = -\frac{\cot x \csc^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2} x dx$

7) $\int \sin^m x \cos^n x dx = -\frac{1}{m+n}\sin^{m-1} x \cos^{n+1} x + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x dx$

8) $\int \frac{\cos^n x}{\sin^m x} dx = -\frac{\cos^{n+1} x}{(m-1)\sin^{m-1} x} - \frac{n-m+2}{m-1}\int \frac{\cos^n x}{\sin^{m-2} x} dx$

9) $\int \frac{\sin^m x}{\cos^n x} dx = -\frac{\sin^{m+1} x}{(n-1)\cos^{n-1} x} - \frac{m-n+2}{n-1}\int \frac{\sin^m x}{\cos^{n-2} x} dx$

10) $\int \cos^m x \cos nx dx = \frac{\cos^m x \sin nx}{m+n} + \frac{m}{m+n}\int \cos^{m-1} x \cos(n-1) x dx$

11) $\int \sin^m x \sin nx dx = \frac{n \sin^m x \cos nx}{m^2 - n^2} - \frac{m}{m^2 - n^2}\sin^{m-1} x \cos x \sin nx + \frac{m(m-1)}{m^2 - n^2}\int \sin^{m-2} x \sin nx dx$

12) $\int \cos^m x \sin nx dx = \frac{1}{m+n}(-\cos^m x \cos nx) + \frac{m}{m+n}\int \cos^{m-1} x \sin(n-1) dx$

13) $\int \sin^m x \cos nx dx = \frac{m \cos x \cos nx + n \sin x \sin nx}{n^2 - m^2}\sin^{m-1} x - \frac{m(m-1)}{n^2 - m^2}\int \sin^{m-2} x \cos nx dx$

14) $\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1} e^{ax} dx$

15) $$\int x^m \sin nx\,dx = -\frac{x^m \cos nx}{n} + \frac{m}{n^2} x^{m-1} \sin nx - \frac{m(m-1)}{n^2} \int x^{n-2} \sin nx\,dx$$

16) $$\int x^m \sin nx\,dx = -\frac{x^m \sin nx}{n} + \frac{m}{n^2} x^{m-1} \cos nx - \frac{m(m-1)}{n^2} \int x^{m-2} \cos nx\,dx$$

17) $$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \begin{cases} \dfrac{(n-1)(n-3)(n-5)\cdots 2}{n(n-2)(n-4)\cdots 3}, & \text{if } n \text{ is odd} \\ \dfrac{(n-1)(n-3)(n-5)\cdots 2}{n(n-2)(n-4)\cdots 2} \cdot \dfrac{\pi}{2}, & \text{if } n \text{ is even} \end{cases}$$

18) $$\int_0^{\frac{\pi}{2}} \cos^n x\,dx = \begin{cases} \dfrac{(n-1)(n-3)(n-5)\cdots 2}{n(n-2)(n-4)\cdots 3}, & \text{if } n \text{ is odd} \\ \dfrac{(n-1)(n-3)(n-5)\cdots 2}{n(n-2)(n-4)\cdots 2} \cdot \dfrac{\pi}{2}, & \text{if } n \text{ is even} \end{cases}$$

The formulas **(17)** and **(18)** are called **Wallis Formulas**.

Results and Formulas of Definite Integrals

1) $$\int_a^b F'(x)dx = F(a) - F(b)$$ Which is called **Fundamental Theorem of Integral Calculus**.

2) $$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

3) $$\int_a^b f(x)dx = \int_a^b f(t)dt$$

4) $$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \quad a < c < b$$

5) $$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

6) $$\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$$

7) If $f(2a-x) = f(x)$ then $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$

8) If $f(2a-x) = -f(x)$ then $\int_0^{2a} f(x)dx = 0$

9) If $f(x)=f(a+x)$ then $\int_0^{na} f(x)dx = n\int_0^a f(x)dx$

10) $\int_0^{\frac{\pi}{2}} \ln(\sin x)dx = \int_0^{\frac{\pi}{2}} \ln(\cos x)dx = -\frac{\pi}{2}\ln 2 = \frac{\pi}{2}\ln\frac{1}{2}$

11) If $f(-x)=f(x)$ i.e. $f(x)$ is an even function, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

12) If $f(-x)=-f(x)$ i.e. $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

13) If $f(x)$ is a periodic function with period p, i.e. $f(x+p)=f(x)$

then for an integer n, $\int_a^{a+np} f(x)dx = n\int_a^b f(x)dx$

14) $\int_0^{\frac{\pi}{2}} f(\sin x)dx = \int_0^{\frac{\pi}{2}} f(\cos x)dx$

15) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

16) $\int_a^b f(x)dx = \int_0^{b-a} f(x+a)dx$

17) $\int_0^{\frac{\pi}{2}} \ln\tan x dx = \int_0^{\frac{\pi}{2}} \ln\cot x dx = 0$

18) $\int_0^{\frac{\pi}{2}} \ln\sec x dx = \int_0^{\frac{\pi}{2}} \ln\csc x dx = \frac{\pi}{2}\ln 2 = -\frac{\pi}{2}\ln\frac{1}{2}$

Factors and Products Formulas

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a+b)^2 = (a-b)^2 + 4ab$
4. $(a-b)^2 = (a+b)^2 - 4ab$

5. $(a+b)^2+(a-b)^2=2a^2+2b^2$

6. $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ac$

7. $(a+b+c+\cdots)^2=a^2+b^2+c^2+\cdots+2(ab+ac+bc+\cdots)$

8. $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$=a^3+b^3+3ab(a+b)$

9. $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

$=a^3-b^3-3ab(a-b)$

10. $(a+b)(a-b)=a^2-b^2$

11. $a^3-b^3=(a-b)(a^2+ab+b^2)$

12. $a^3+b^3=(a+b)(a^2-ab+b^2)$

13. $(a+b)(a+c)=a^2+(b+c)a+bc$

$(x+b)(x+c)=x^2+(b+c)x+bc$

14. $(a+b+c)(a^2+b^2+c^2-ac-bc-ca)=a^3+b^3+c^3-3abc$

15. $a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\cdots+b^{n-1})$ if n is odd.

16. $a^n-b^n=(a+b)(a^{n-1}-a^{n-2}b+a^{n-3}b^2-\cdots-b^{n-1})$ if n is even.

17. $a^n+b^n=(a+b)(a^{n-1}-a^{n-2}b+a^{n-3}b^2-\cdots-b^{n-1})$ if n is odd.

18. $(x+a)(x+b)(x+c)=x^3+(a+b+c)x^2+(ab+bc+ac)x+abc$

Improtant Math Series

1. $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$

2. $\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots+(-1)^{n-1}\frac{x^n}{n}+\cdots$

3. $\ln(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-\cdots-\frac{x^n}{n}-\cdots$

4. $a^x=1+\frac{x\ln a}{1!}+\frac{(x\ln a)^2}{2!}+\frac{(x\ln a)^3}{3!}+\cdots$

5. $Sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!} + \cdots$

6. $Cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots$

7. $Tanx = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$

8. $Secx = 1 + \frac{1}{2}x^2 + \frac{5}{4!}x^4 + \frac{16}{6!}x^6 + \cdots$

9. $e^{Sinx} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8} - \frac{x^5}{15} - \cdots$

10. $Sin^{-1}x = \frac{x}{1} + \frac{1}{2}\frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4}\frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\frac{x^7}{7} + \cdots$

11. $(1+x)^{\frac{1}{x}} = e(1 - \frac{1}{2}x + \frac{11}{24}x^2 + \cdots)$

12. $(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{1 \cdot 2}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \cdots$

13. $(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{1 \cdot 2}{2 \cdot 4}x^2 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \cdots$

14. $(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 + \cdots$

15. $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

16. $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$

Exponents Formulas

Exponents

1. If p is positive integer, and $a \in \mathbb{R}$, then $a^p = a \cdot a \cdot a \cdots$ to p factors.
2. $\forall a \in \mathbb{R}$, $a \neq 0$, $a^0 = 1$

3. $a^r \cdot a^s = a^{r+s}$, $a \in \mathbb{R}$; $r, s \in \mathbb{N}; a \neq 0$

4. $(a^r)^s = a^{rs}$

5. $(ab)^r = a^r \cdot b^r$ $a \in \mathbb{R}$; $r \in \mathbb{N}; a, b \neq 0$

6. $1^n = 1$ $\forall n \in \mathbb{N}$

7. $\frac{a^r}{a^s} = a^{r-s}$, $a \in \mathbb{R}$; $r, s \in \mathbb{N}; a \neq 0$

8. $(\frac{a}{b})^r = \frac{a^r}{b^r}$ $a \in \mathbb{R}$; $r \in \mathbb{N}; a, b \neq 0$

9. $a^{-r} = \frac{1}{a^r}$ $a \in \mathbb{R}$; $r \in \mathbb{N}; a \neq 0$

10. $a^{\frac{r}{s}} = \sqrt[s]{a^r}$ $a \in \mathbb{R}$; $r, s \in \mathbb{N}; a \neq 0$

Logarithm Formulas

Logarithms

1. $y = \log_a x$ if and only if $x = a^y$, $x > 0$ and $y \in \mathbb{R}$, a is positive no.
2. $\log_a 1 = 0$
3. $\log_a a = 1$, $\log_e e = 1$, i.e. $\ln e = 1$
4. $\log_a mn = \log_a m + \log_a n$
5. $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
6. $\log_a (m)^n = n \log_a m$
7. $\log_a m = \log_b m \cdot \log_a b$
8. $\log_b m = \frac{\log_a m}{\log_a b}$
9. $\log_b a = \frac{1}{\log_a b}$
10. $\log_a x = \frac{\ln x}{\ln a} = (\log_a e) \ln x$
11. $\log \frac{bc}{a^2} + \log \frac{ac}{b^2} + \log \frac{ab}{c^2} = 0$

12. $\log n! = \log 2 + \log 3 + \log 4 + \cdots + \log n$

Math Series Results

1. $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
2. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
3. $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$
4. $1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
5. $2 + 4 + 6 + \cdots + 2n = n(n+1)$
6. $1 + 3 + 5 + \cdots + (2n-1) = n^2$
7. $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6} = 1.64493$
8. $1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots = 1.20205$
9. $1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90} = 1.08232$
10. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \log_e 2 = 0.6931$
11. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$
12. $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \pi^2$
13. $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$
14. $1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots = e$

Formulas of Sequence and Series

1. The nth term a_n of the Arithmetic Progression (A.P) $a,\ a+d,\ a+2d, \ldots$ is given by $a_n = a + (n-1)d$.

2. Arithmetic mean between a and b is given by $A.M = \frac{a+b}{2}$.

3. If S_n denotes the sum up to n terms of A.P. $a,\ a+d,\ a+2d, \ldots$ then $S_n = \frac{n}{2}(a+l)$ where l stands for last term, $S_n = \frac{n}{2}[2a+(n-1)d]$

4. The sum of n A.M's between a and b is $= \frac{n(a+b)}{2}$.

5. The nth term a_n of the geometric progression $a,\ ar,\ ar^2,\ ar^3, \ldots$ is $a_n = ar^{n-1}$.

6. Geometric mean between a and b is $G.M = \pm\sqrt{ab}$.

7. If S_n denotes the sum up to n terms of G.P is $S_n = \frac{a(1-r^n)}{1-r};\ \ r \neq 1$, $S_n = \frac{a-rl}{1-r};\ \ l = ar^n$ where $|r| < 1$

8. The sum S of infinite geometric series is $S = \frac{a}{1-r};\ \ |r| < 1$

9. The nth term a_n of the geometric progression is $a_n = \frac{1}{a+(n-1)d}$.

10. Harmonic mean between a and b is $H.M = \frac{2ab}{a+b}$.

11. $G^2 = A \cdot H$ and $A > G > H$; where A, G, H are usual notations.

Formulas and Results of Complex Numbers

1. $z = (a,b) = a+ib,\ \ i = (0,1)$

2. $i = \sqrt{-1},\ \ i^2 = -1,\ \ i^3 = -i,\ \ i^4 = 1,\ \ i^5 = i,\ \ldots$

3. If n is a positive integer then, $(i)^{4n} = 1,\ \ (i)^{4n+1} = i,\ \ (i)^{4n+2} = -1,\ \ (i)^{4n+3} = -i$

4. If $a+ib = 0$ then $a = b = 0$ and conversely.

5. If $a+ib = c+id$ then $a = c$ and $b = d$

6. $(a,b)+(c,d) = (a+c,\ b+d)$

7. $(a,b)(c,d) = (ac-bd,\ ad+bc)$

8. $z_1 + z_2 = z_2 + z_1\ ;\ \forall z_1, z_2 \in \mathbb{C}$

9. $z_1 \cdot z_2 = z_2 \cdot z_1\ ;\ \forall z_1, z_2 \in \mathbb{C}$

10. $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3\ ;\ \forall z_1, z_2, z_3 \in \mathbb{C}$

11. $z_1 \cdot (z_2 \cdot z_3) = (z_1 \cdot z_2) \cdot z_3\ ;\ \forall z_1, z_2, z_3 \in \mathbb{C}$

12. $(0,0)$ is the additive identity.

13. $(1,0)$ is the multiplicative identity.

14. If $z = a + ib$ the multiplicative inverse of z is $z^{-1} = \frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2}$

15. Additive inverse of z is $-z = -a - ib$

16. If $z = a + ib$, then $\bar{z} = a - ib$

17. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2 \;;\; \forall z_1, z_2 \in \mathbb{C}$

18. $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2 \;;\; \forall z_1, z_2 \in \mathbb{C}$

19. $\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2 \;;\; \forall z_1, z_2 \in \mathbb{C}$

20. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2} \;;\quad \forall z_1, z_2 \in \mathbb{C}$

21. If $\bar{z} = z$, then z is real number.

22. $\overline{(\bar{z})} = z$

23. If $z = a + ib$, $a = \mathrm{Re}(z),\ b = \mathrm{Im}(z)$

24. $z\,\bar{z} = (\mathrm{Re}\,z)^2 + (\mathrm{Im}\,z)^2$

25. If $z = a + ib$, the $|z| = \sqrt{a^2 + b^2}$

26. $|z| \geq 0$

27. $|z| = |-z| = |\bar{z}|$

28. $|z|^2 = z\,\bar{z}$

29. $|z_1 z_2| = |z_1||z_2|$

30. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|},\ z_2 \neq 0$

31. $|z_1| - |z_2| \leq |z_1 + z_2| \leq |z_1| + |z_2|$

32. $|z_1 - z_2| \geq |z_1| - |z_2|$

33. $|\mathrm{Re}\,z| \leq |z|$, $|\mathrm{Im}\,z| \leq |z|$

34. $|z_1 - z_2| = |z_2 - z_1|$

35. $||z_1| - |z_2|| \leq |z_1 - z_2|$

36. $z = r(Cos\theta + iSin\theta)$ is polar form of z, where $r = |z| \;;\; \theta = Tan^{-1}\left(\frac{b}{a}\right) = \arg(z)$

37. If $z_1 = r_1(Cos\theta_1 + iSin\theta_1)$ and $z_2 = r_2(Cos\theta_2 + iSin\theta_2)$, then

- $z_1 \; z_2 = r_1 r_2[Cos(\theta_1 + \theta_2) + iSin(\theta_1 + \theta_2)]$
- $\frac{z_1}{z_2} = \frac{r_1}{r_2}[Cos(\theta_1 - \theta_2) + iSin(\theta_1 - \theta_2)]$
- $\arg(z_1 z_2) = \arg z_1 + \arg z_2$
- $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

1. $CiS\theta = Cos\theta + iSin\theta = e^{i\theta}$
2. $(z)^0 = 1$
3. $(z)^{m+1} = z^m z$
4. $(z)^{-m} = (z^{-1})^m, \qquad m \in \mathbb{Z}^+$
5. $(z^m)^n = (z)^{mn}$
6. $(z_1 z_2)^n = (z_1)^n (z_2)^n$
7. $(Cos\theta + iSin\theta)^n = Cos\; n\theta + iSin\; n\theta$. For all integer n, is called De-Moivre's Theorem.

Formulas of Useful Limits

1) If $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} g(x) = m$, then

- $\lim_{x \to a}[f(x) \pm g(x)] = l \pm m$
- $\lim_{x \to a} f(x) \cdot g(x) = l \cdot m$
- $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{l}{m}$, Where $m \neq 0$
- $\lim_{x \to a} c\, f(x) = c\, l$
- $\lim_{x \to a} \frac{1}{f(x)} = \frac{1}{l}$, Where $l \neq 0$

2) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$, Where n is a real number.

3) $\lim_{n\to 0}(1+n)^{\frac{1}{n}} = e$, Where n is a real number.

4) $\lim_{x\to 0}\frac{Sinx}{x} = 1$, Where x is measured in radians.

5) $\lim_{x\to 0}\frac{Tanx}{x} = 1$

6) $\lim_{x\to 0}\frac{Cosx-1}{x} = 0$

7) $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$

8) $\lim_{x\to 0}\frac{a^x-1}{x} = \ln a$

Set Theory Formula

Consider that A, B and C are the sets, then

1. $A \cup A = A$
2. $A \cap A = A$ are called Idempotent laws.
3. $A \cup B = B \cup A$
4. $A \cap B = B \cap A$ are called Commutative laws.
5. $(A \cup B) \cup C = A \cup (B \cup C)$
6. $(A \cup B) \cup C = A \cup (B \cup C)$ are called Associative laws.
7. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
8. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ are called Distributive laws.
9. $(A \cup B)^C = A^C \cap B^C$
10. $(A \cap B)^C = A^C \cup B^C$ are called De-Morgan's Laws.
11. $A - (B \cup C) = (A - B) \cap (A - C)$
12. $A - (B \cap C) = (A - B) \cup (A - C)$
13. $A - (B \cup C) = A \cap (B \cup C)^C$

14. $A \cap (B - C) = (A \cap B) - C$

15. $A \Delta B = (A - B) \cup (B - A)$ is called Symmetric Difference.

16. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

17. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

18. $A \times (B - C) = (A \times B) - (A \times C)$

Set Theory Results

Consider that A, B and C are the sets, then

1. $A \subseteq A \cup B$ and $B \subseteq A \cup B$
2. $A \cup \varnothing = A$
3. $A \cap \varnothing = \varnothing$
4. $A \subseteq C$ and $B \subseteq C$ then $A \cup B \subseteq C$
5. $A \subset B$ if and only if $A \cup B = B$
6. $A \subset B$ if and only if $A \cap B = A$
7. $A \cap B \subseteq A$ and $A \cap B \subseteq B$
8. If $A \subseteq B$ and $A \subseteq C$ then $A \subseteq B \cap C$
9. $A \cap U = A$
10. $A \cup U = U$
11. $(A^c)^c = A$
12. $\varnothing^c = U$ and $U^c = \varnothing$
13. $A \cup A^c = U$
14. $A \cap A^c = \varnothing$
15. If $A \subseteq B$ then $B^c \subseteq A^c$
16. If $x \in A$ and $x \in B$ then $x \in A \cap B$
17. If $x \in A$ or $x \in B$ then $x \in A \cup B$
18. $A - B = A \cap B^c$
19. $A - (A - B) = A \cap B$
20. $A - (A - B) = A - (A \cap B^c)$

Fundamental Trigonometric Ratios

1) $Sin\theta = \frac{perpendicular}{hypotenuse} = \frac{y}{r}$

2) $Cos\theta = \frac{base}{hypotenuse} = \frac{x}{r}$

3) $Tan\theta = \frac{perpendicular}{base} = \frac{y}{x} = \frac{Sin\theta}{Cos\theta}$

4) $Cot\theta = \frac{base}{perpendicular} = \frac{x}{y} = \frac{Cos\theta}{Sin\theta}$

5) $Sec\theta = \frac{hypotenuse}{base} = \frac{r}{x} = \frac{1}{Cos\theta}$

6) $Cosec\,\theta = \frac{hypotenuse}{perpendicular} = \frac{r}{y} = \frac{1}{Sin\theta}$

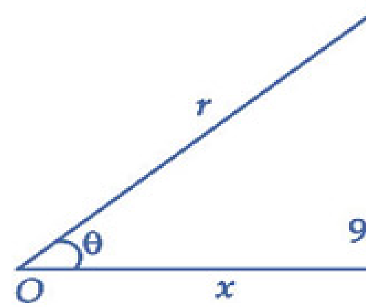
7) Signs of trigonometric Ratios

I QUADRANT: A, means all trigonometric ratios are positive.
II QUADRANT: S, means Sin and $Cosec$ are positive all others are negative.
III QUADRANT: T, means Tan and Cot are positive all others are negative.
IV QUADRANT: C, means Cos and Sec are positive all others are negative.

NOTE: (1) Clockwise; we read $ACTS$

(2) Anticlockwise; we read $ASTC$ (All Silver Tea Cups)

θ in Quadrant	$Sin\theta$	$Cos\theta$	$Tan\theta$	$Cot\theta$	$Sec\theta$	$Cosec\,\theta$
I	+	+	+	+	+	+
II	+	−	−	−	−	+
III	−	−	+	+	−	−
IV	−	+	−	−	+	−



8) Trigonometric Ratios of Special Angles:

θ	$Sin\theta$	$Cos\theta$	$Tan\theta$	$Cot\theta$	$Sec\theta$	$Cosec\,\theta$
$0°$	0	1	0	∞	1	∞
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
$90°$	1	0	∞	0	∞	1

RULE: Write $0, 1, 2, 3, 4$ divide by 4 then taking square roots and write the resulting number in column of $Sin\theta$.

Results of Trigonometric Ratios of Allied Angles

α	$Sin\alpha$	$Cos\alpha$	$Tan\alpha$	$Cot\alpha$	$Sec\alpha$	$Cosec\,\alpha$
$-\theta$	$-Sin\theta$	$+Cos\theta$	$-Tan\theta$	$-Cot\theta$	$+Sec\theta$	$-Cosec\,\theta$
$90^\circ-\theta$	$+Cos\theta$	$+Sin\theta$	$+Cot\theta$	$+Tan\theta$	$+Cosec\,\theta$	$+Sec\theta$
$90^\circ+\theta$	$+Cos\theta$	$-Sin\theta$	$-Cot\theta$	$-Tan\theta$	$-Cosec\,\theta$	$+Sec\theta$
$180^\circ-\theta$	$+Sin\theta$		$-Tan\theta$	$-Cot\theta$	$-Sec\theta$	$+Cosec\,\theta$
$180^\circ+\theta$	$-Sin\theta$	$-Cos\theta$	$+Tan\theta$	$+Cot\theta$	$-Sec\theta$	$-Cosec\,\theta$
$270^\circ-\theta$	$-Cos\theta$	$-Sin\theta$	$+Cot\theta$	$+Tan\theta$	$-Cosec\,\theta$	$-Sec\theta$
$270^\circ+\theta$	$-Cos\theta$	$+Sin\theta$	$-Cot\theta$	$-Tan\theta$	$+Cosec\,\theta$	$-Sec\theta$
$360^\circ-\theta$	$-Sin\theta$	$+Cos\theta$	$-Tan\theta$	$-Cot\theta$	$+Sec\theta$	$-Cosec\,\theta$
$360^\circ+\theta$	$+Sin\theta$	$+Cos\theta$	$+Tan\theta$	$+Cot\theta$	$+Sec\theta$	$+Cosec\,\theta$

22) Period of $Sin\theta$ and $Cos\theta$ is 2π, where as period of $Tan\theta$ and $Cot\theta$ is π.

If k is any integer, then

23) $Sin(k\pi)=0$

24) $Cos(k\pi)=(-1)^k$

25) $Sin(k\pi+\beta)=(-1)^k Sin\beta$

26) $Cos(k\pi+\beta)=(-1)^k Cos\beta$

27) $Sin\left[(2k+1)\frac{\pi}{2}+\beta\right]=(-1)^k Cos\beta$

28) $Cos\left[(2k+1)\frac{\pi}{2}+\beta\right]=(-1)^{k+1}Sin\beta$

Fundamental General Identites Involving Trigonometric Ratios

1. $Sin^2\theta + Cos^2\theta = 1$
2. $1 + Tan^2\theta = Sec^2\theta$
3. $1 + Cot^2\theta = Cosec^2\theta$
4. $Sin(\alpha + \beta) = Sin\alpha Cos\beta + Cos\alpha Sin\beta$
5. $Sin(\alpha - \beta) = Sin\alpha Cos\beta - Cos\alpha Sin\beta$
6. $Cos(\alpha + \beta) = Cos\alpha Cos\beta - Sin\alpha Sin\beta$
7. $Cos(\alpha - \beta) = Cos\alpha Cos\beta + Sin\alpha Sin\beta$
8. $Tan(\alpha + \beta) = \frac{Tan\alpha + Tan\beta}{1 - Tan\alpha Tan\beta}$
9. $Tan(\alpha - \beta) = \frac{Tan\alpha - Tan\beta}{1 + Tan\alpha Tan\beta}$
10. $Tan(45^\circ + \theta) = \frac{1 + Tan\theta}{1 - Tan\theta}$
11. $Tan(45^\circ - \theta) = \frac{1 - Tan\theta}{1 + Tan\theta}$
12. $Cot(\alpha + \beta) = \frac{Cot\alpha Cot\beta - 1}{Cot\alpha + Cot\beta}$
13. $Cot(\alpha - \beta) = \frac{Cot\alpha Cot\beta + 1}{Cot\alpha - Cot\beta}$
14. $Sin2\theta = 2Sin\theta Cos\theta$
15. $Cos2\theta = Cos^2\theta - Sin^2\theta$
16. $Cos2\theta = 1 - 2Sin^2\theta$
17. $Cos2\theta = 2Cos^2\theta - 1$
18. $Tan2\theta = \frac{2Tan\theta}{1 - Tan^2\theta} = \frac{2Cot\theta}{Cot^2\theta - 1}$
19. $Cot2\theta = \frac{Cot^2\theta - 1}{2Cot\theta} = \frac{Cot\theta - Tan\theta}{2} = \frac{1 - Tan^2\theta}{2Tan\theta}$
20. $Sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - Cos\theta}{2}}$
21. $Cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + Cos\theta}{2}}$

22. $1 - Cosn\theta = 2Sin^2 \frac{n\theta}{2}$

23. $1 + Cosn\theta = 2Cos^2 \frac{n\theta}{2}$

24. $Tan\frac{\theta}{2} = \frac{1 - Cos\theta}{Sin\theta} = \frac{Sin\theta}{1 + Cos\theta} = \pm\sqrt{\frac{1 - Cos\theta}{1 + Cos\theta}}$

25. $Sin3\theta = 3Sin\theta - 4Sin^3\theta$

26. $Cos3\theta = 4Cos^3\theta - 3Cos\theta$

27. $Tan3\theta = \frac{3Tan\theta - Tan^3\theta}{1 - 3Tan^2\theta}$

28. $Cot3\theta = \frac{Cot^3\theta - 3Cot\theta}{3Cot^2\theta - 1}$

Formulas of Trigonometric and Logarithmic Functions

1) $Sinx = \frac{e^{ix} - e^{-ix}}{2i}$

2) $Cosx = \frac{e^{ix} + e^{-ix}}{2}$

3) $Tanx = \frac{e^{ix} - e^{-ix}}{i(e^{ix} + e^{-ix})}$

4) $Cotx = \frac{i(e^{ix} + e^{-ix})}{e^{ix} - e^{-ix}}$

5) $Secx = \frac{2}{e^{ix} + e^{-ix}}$

6) $Cosec\, x = \frac{2i}{e^{ix} - e^{-ix}}$

7) $Log(x + iy) = Log\sqrt{x^2 + y^2} + iTan^{-1}\left(\frac{y}{x}\right)$

8) $Log\left(\frac{x + iy}{x - iy}\right) = 2iTan^{-1}\left(\frac{y}{x}\right)$

Formulas of Sum and Product of Trigonometric Ratios

1. $Sin a + Sin b = 2 Sin\left(\frac{a+b}{2}\right) Cos\left(\frac{a-b}{2}\right)$
2. $Sin a - Sin b = 2 Cos\left(\frac{a+b}{2}\right) Sin\left(\frac{a-b}{2}\right)$
3. $Cos a + Cos b = 2 Cos\left(\frac{a+b}{2}\right) Cos\left(\frac{a-b}{2}\right)$
4. $Cos a - Cos b = -2 Sin\left(\frac{a+b}{2}\right) Sin\left(\frac{a-b}{2}\right)$
5. $2\ Sin a\ Cos b = Sin(a+b) + Sin(a-b)$
6. $2\ Cos a\ Sin b = Sin(a+b) - Sin(a-b)$
7. $2\ Cos a\ Cos b = Cos(a+b) + Cos(a-b)$
8. $2\ Sin a\ Sin b = Cos(a-b) - Cos(a+b)$
9. $Sin^2 a - Sin^2 b = Sin(a+b) Sin(a-b)$
10. $Cos^2 a - Sin^2 b = Cos(a+b) Cos(a-b) = Cos^2 b - Sin^2 a$

Results and Formulas of Plane Triangles

Consider the triangle ΔABC, having the angles α, β, γ and sides a, b, c as shown in the figure.

1) Law of Sine is $\frac{a}{Sin\alpha} = \frac{b}{Sin\beta} = \frac{c}{Sin\gamma}$

2) Laws of Cosine are

$$a^2 = b^2 + c^2 - 2bc Cos\alpha$$

or

$$Cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac Cos\beta$$

or

$$Cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab Cos\gamma$$

or

$$Cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

3) Laws of Tangent are

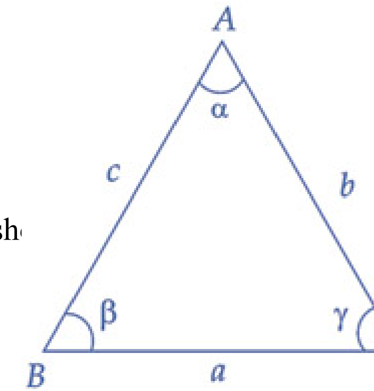
$$\frac{Tan\left(\frac{A-B}{2}\right)}{Tan\left(\frac{A+B}{2}\right)} = \frac{a-b}{a+b}$$

$$\frac{Tan\left(\frac{B-C}{2}\right)}{Tan\left(\frac{B+C}{2}\right)} = \frac{b-c}{b+c}$$

$$\frac{Tan\left(\frac{A-C}{2}\right)}{Tan\left(\frac{A+C}{2}\right)} = \frac{a-c}{a+c}$$

HALF ANGLE FORMULAS

Consider that a, b, c and A, B, C are sides and angles of ΔABC, as sh



(i) $s = \frac{a+b+c}{2}$ (ii) $r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$

4) $Sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

5) $Sin\frac{\beta}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$

6) $Sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

7) $Cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$

8) $Cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ac}}$

9) $Cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$

10) $Tan\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{r}{s-a}$

11) $Tan\frac{\beta}{2} = \sqrt{\frac{(s-a)(s-c)}{s\,(s-b)}} = \frac{r}{s-b}$

12) $Tan\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s\,(s-c)}} = \frac{r}{s-c}$

Formulas of Inverse Trigonometric Functions

1) $Sin^{-1}x = \frac{1}{i}Log(ix + \sqrt{1-x^2})$

2) $Cos^{-1}x = \frac{1}{i}Log(x + \sqrt{x^2-1})$

3) $Tan^{-1}x = \frac{1}{2i}Log\left(\frac{1+ix}{1-ix}\right)$

4) $Cot^{-1}x = \frac{1}{2i}Log\left(\frac{x+i}{x-i}\right)$

5) $Sec^{-1}x = \frac{1}{i}Log\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

6) $Cosec^{-1}x = \frac{1}{i}Log\left(\frac{i+\sqrt{x^2-1}}{x}\right)$

7) $Tan^{-1}\left(\frac{x+iy}{x-iy}\right) = \frac{\pi}{4} + \frac{i}{2}Log\left(\frac{x+y}{x-y}\right)$

Change of Hyperbolic Functions

1) $Sinh\ ix = iSinx$

2) $Cosh\ ix = Cosx$

3) $Tanh\ ix = iTanx$

4) $iSinhx = Sin\ ix$

5) $Coshx = Cosix$

6) $iTanhx = Tan\ ix$